WIRELESS PERSONAL COMMUNICATIONS SYSTEMS

RADIO SIGNAL PROPAGATION

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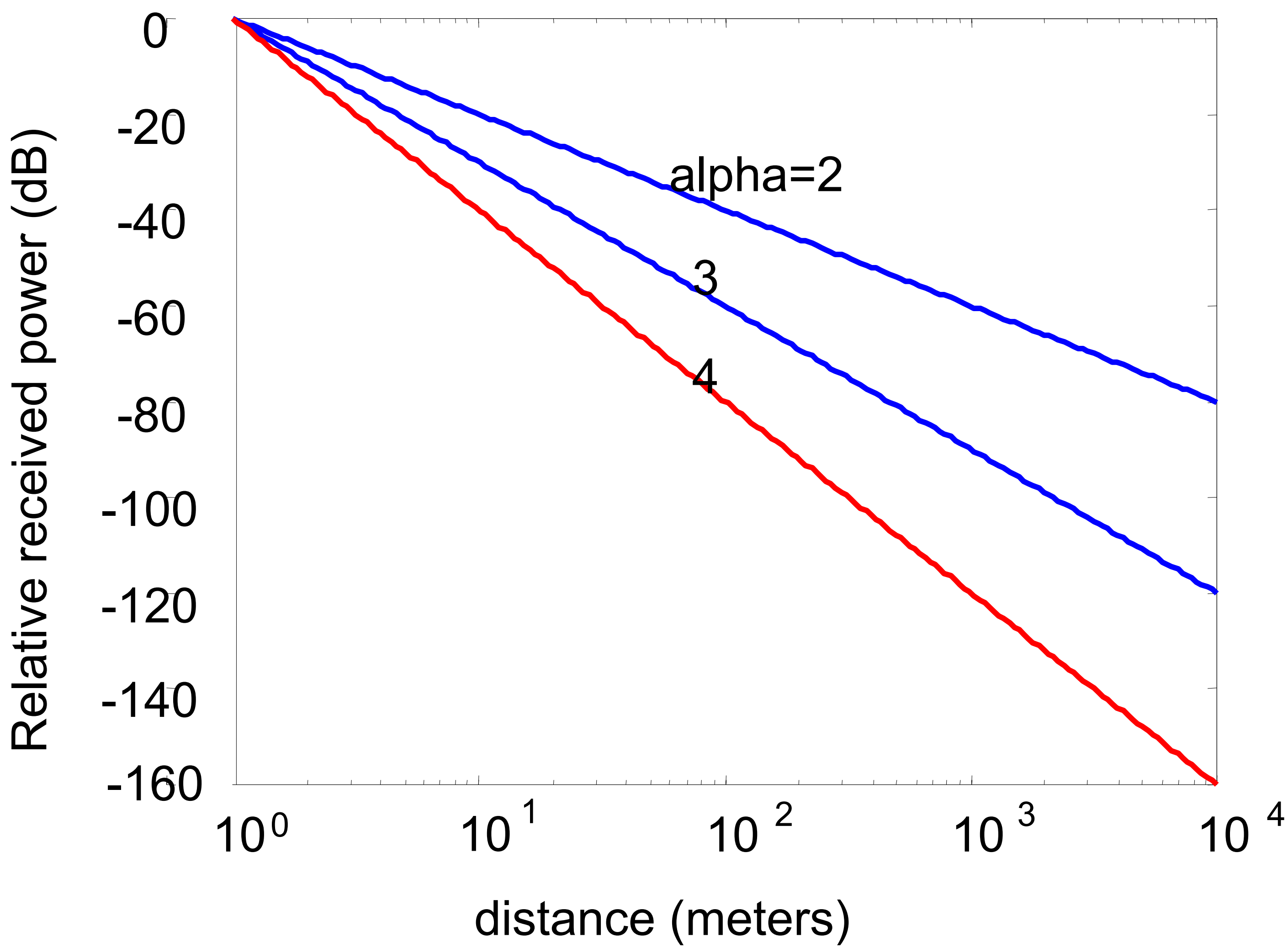
Propagation of a mobile radio signal

- Attenuation
  - distance between transmitter and receiver
- Shadow fading (aka slow fading, log-normal fading)
  - terrain features, buildings (outdoors)
  - building materials, walls, floors, furniture (indoors)
- Rayleigh fading, Rice fading (aka fast fading)
  - motion, velocity
- Multipath
  - terrain features, buildings (outdoors)
  - building materials, walls, floors, furniture (indoors)

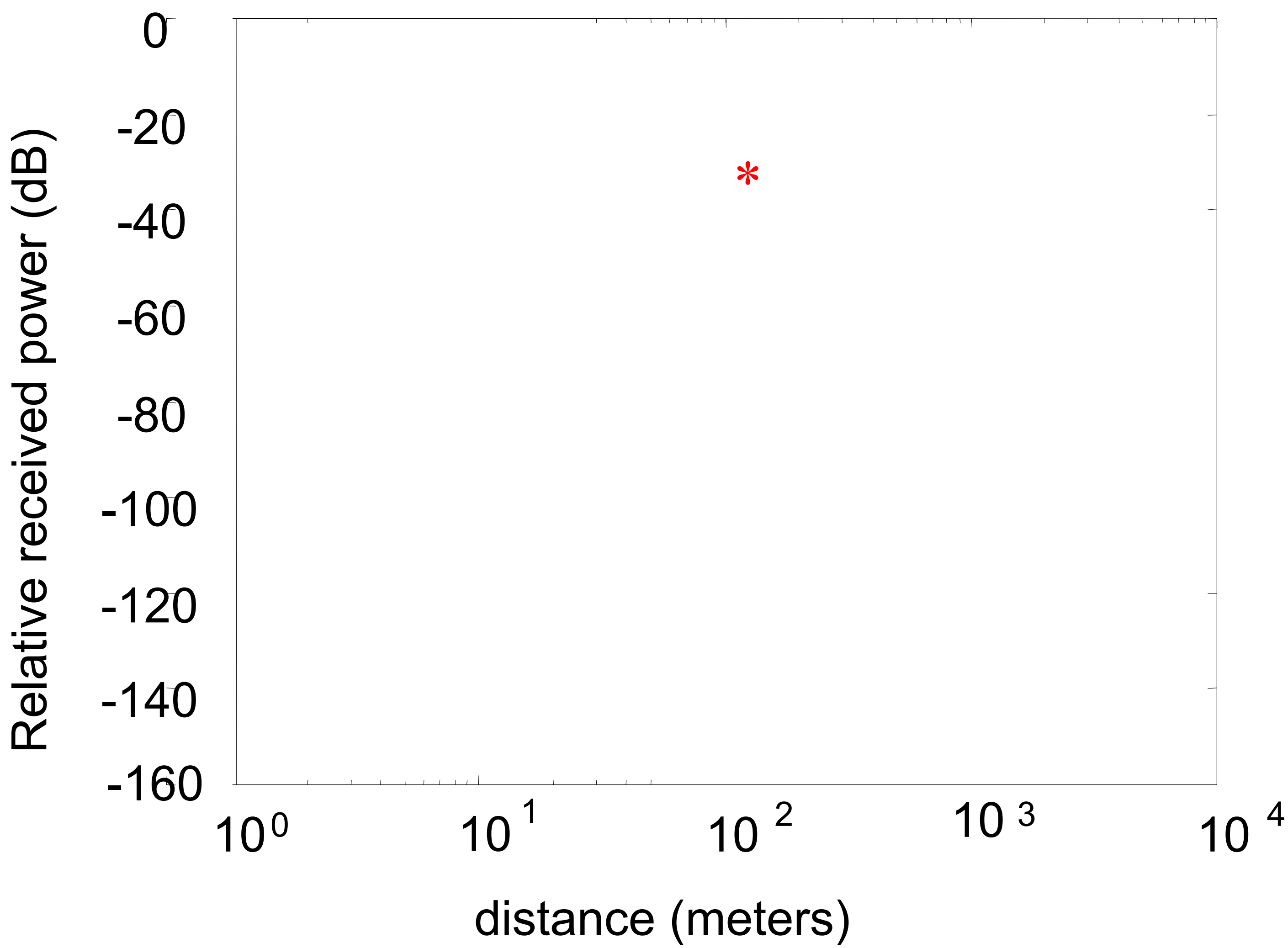
Propagation of a radio signal

- Attenuation — Exponential decrease
  - Propagation exponent α
- Shadow fading: — Normal random variable
  - Average received power (space average) T dBm
  - Standard deviation σ dB
- Rayleigh, Rice fading — Probability model, spectrum
  - Average received power, P watts,
  - Carrier wavelength λ meters,
  - Velocity v m/s
- Multipath — Delay spread
  - Path delays t_i seconds
  - Path attenuations a_i

Attenuation v. distance



Attenuation v. distance



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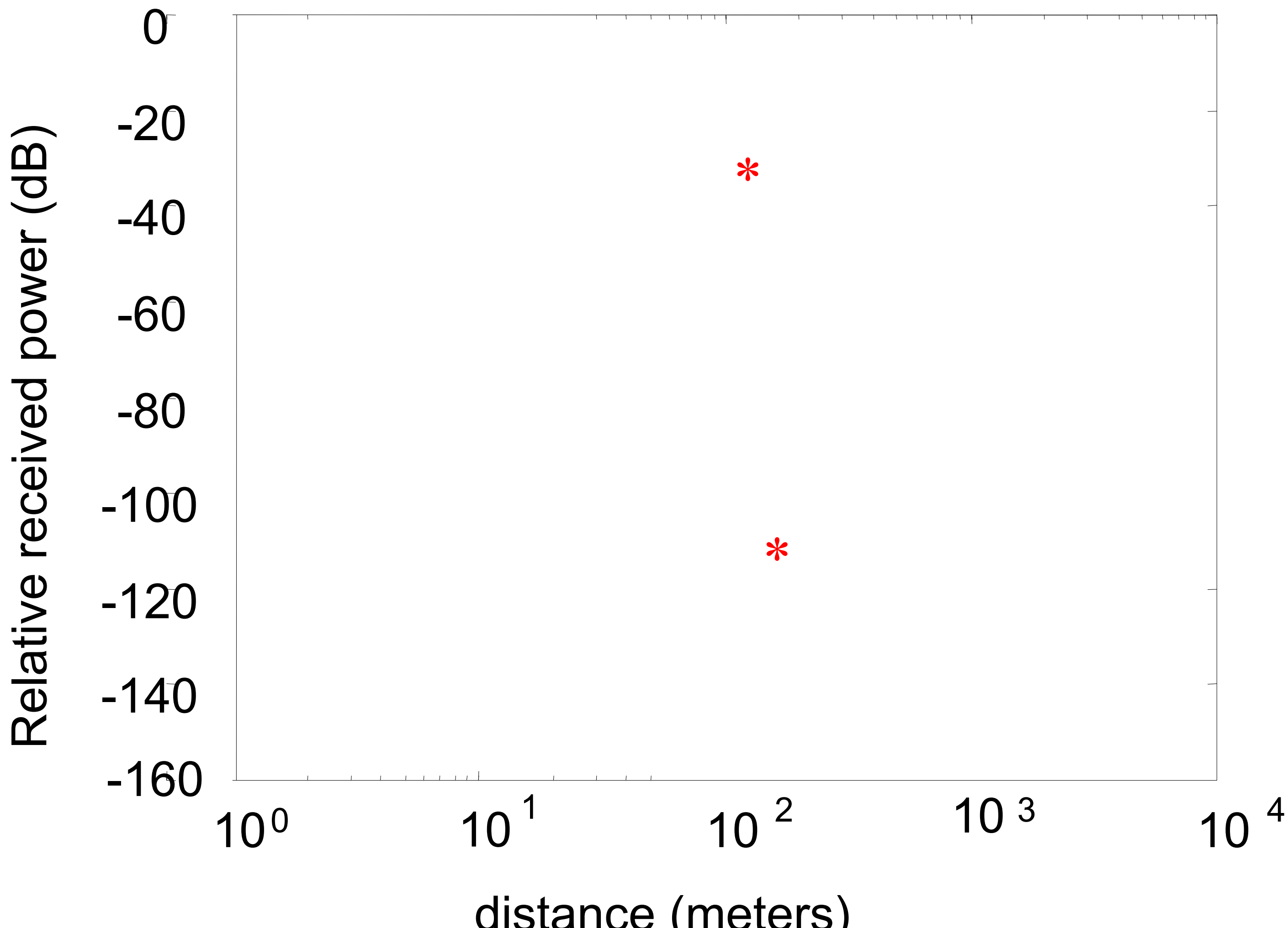
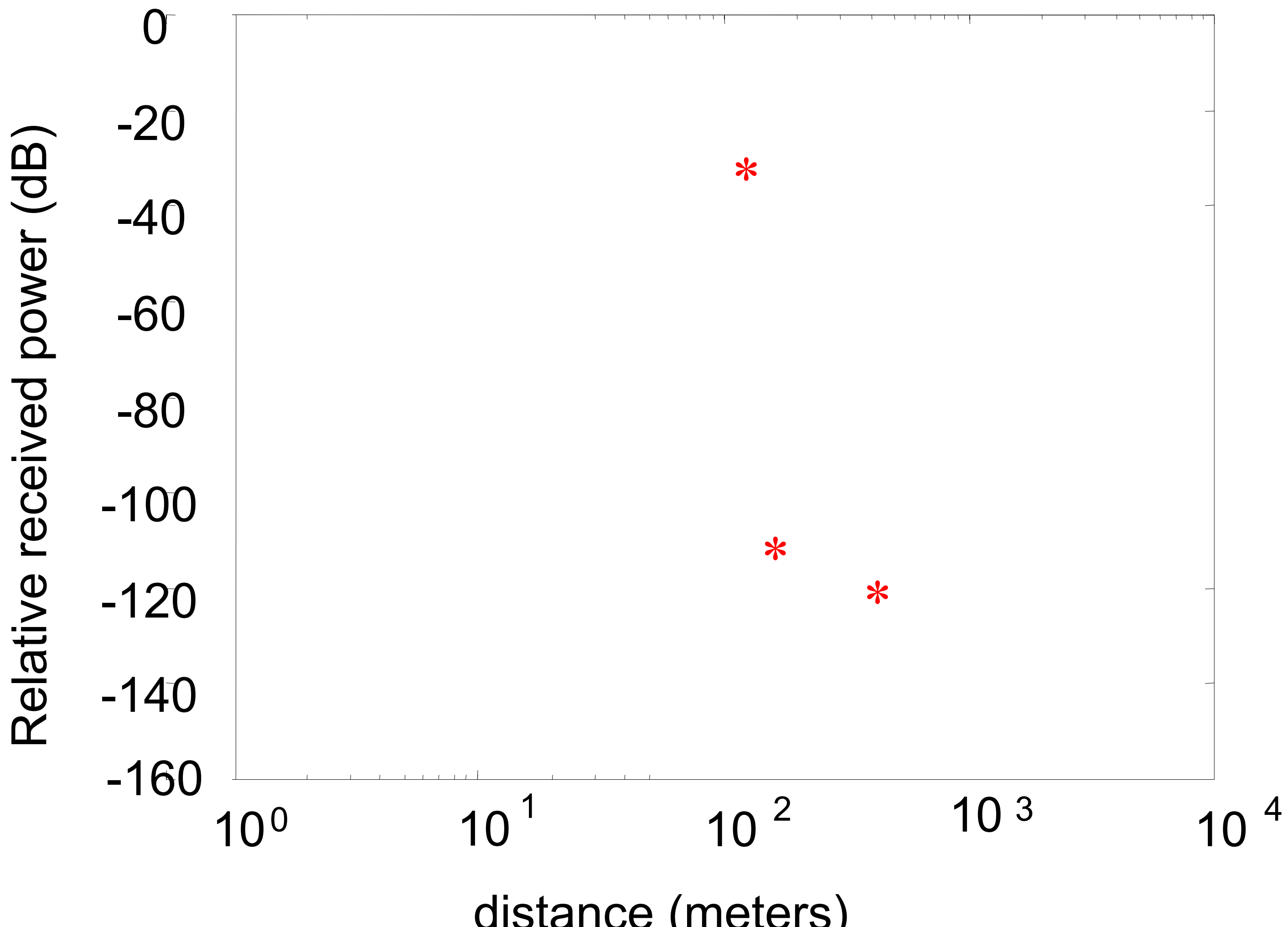
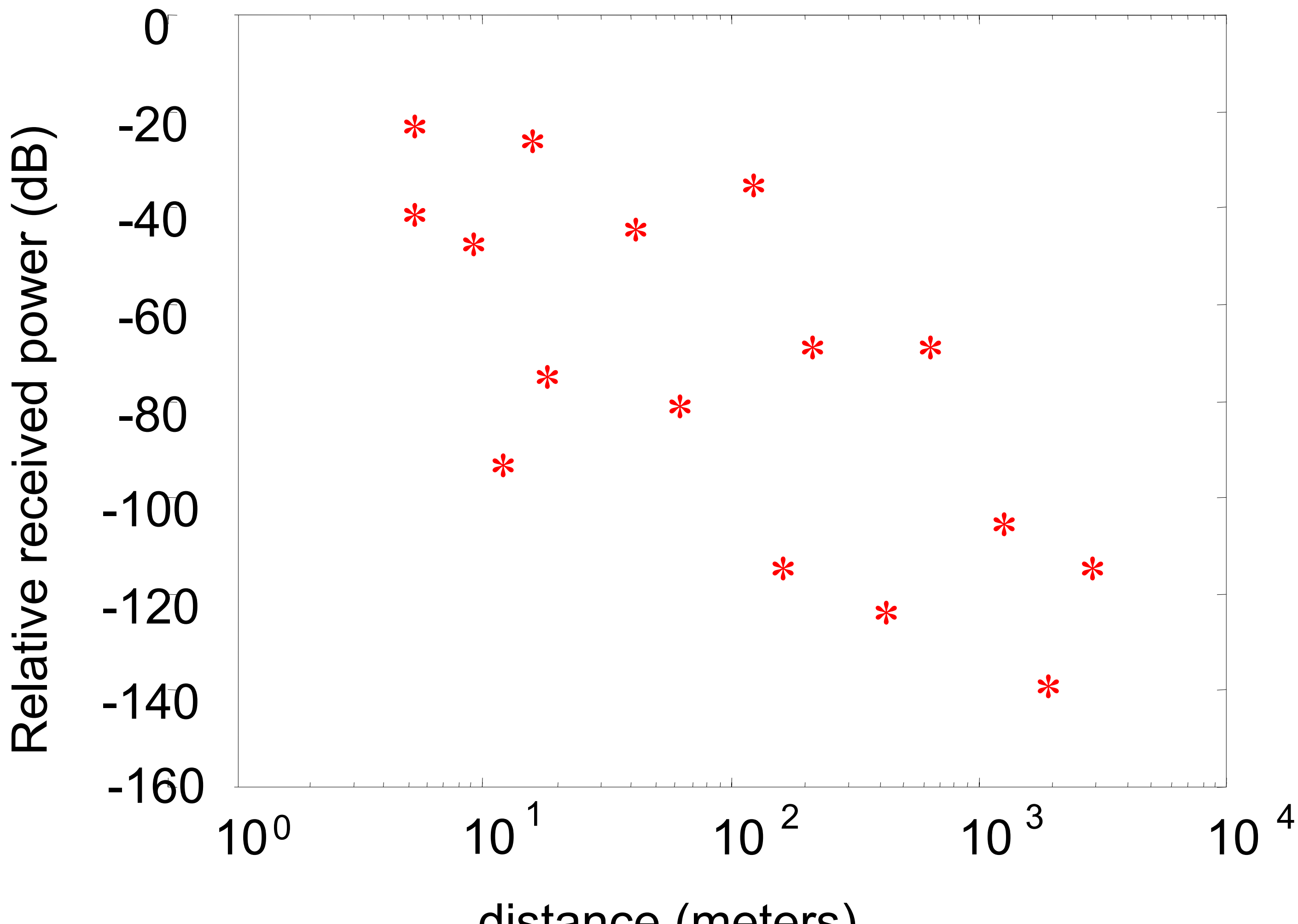


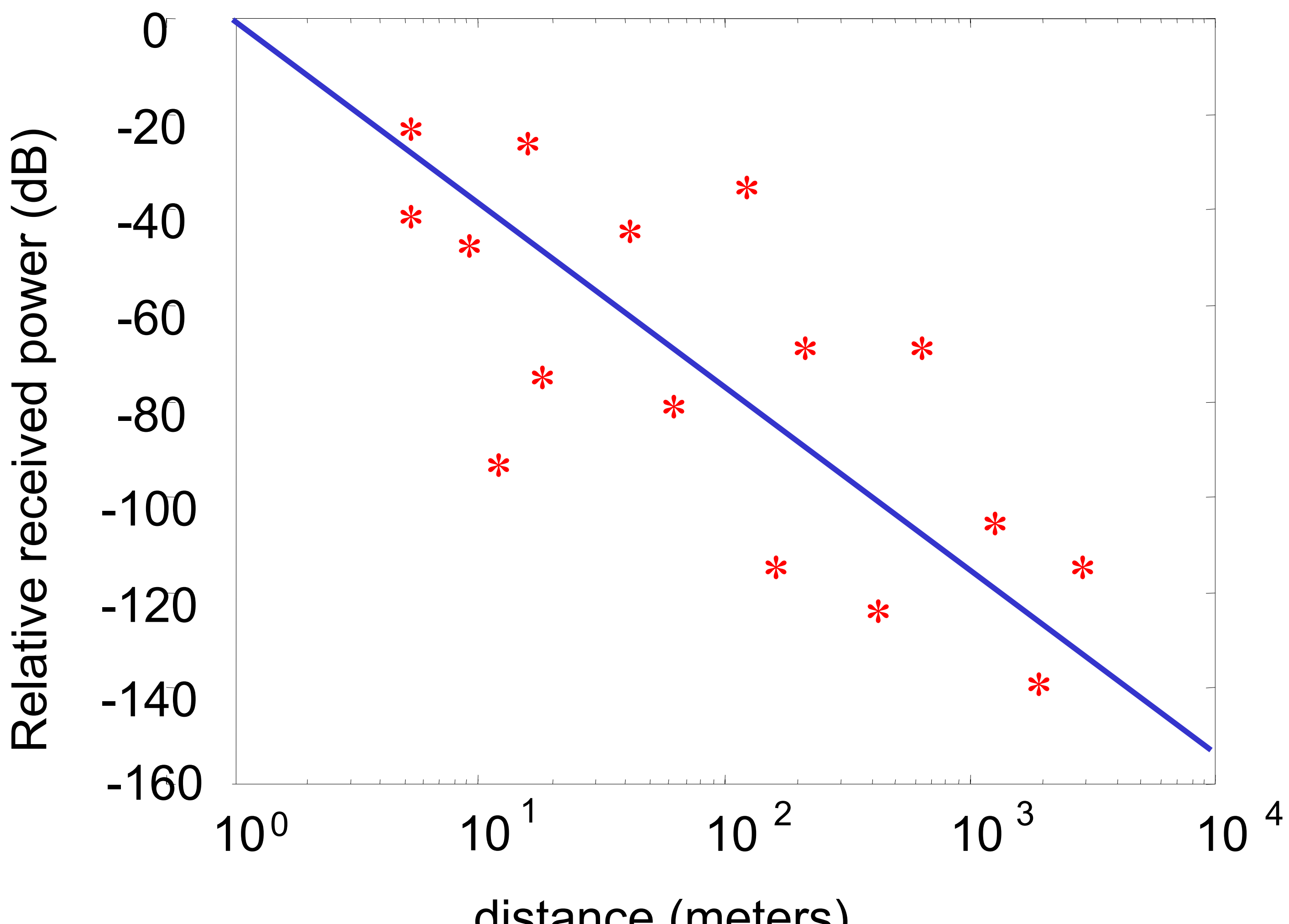
Attenuation v. distance



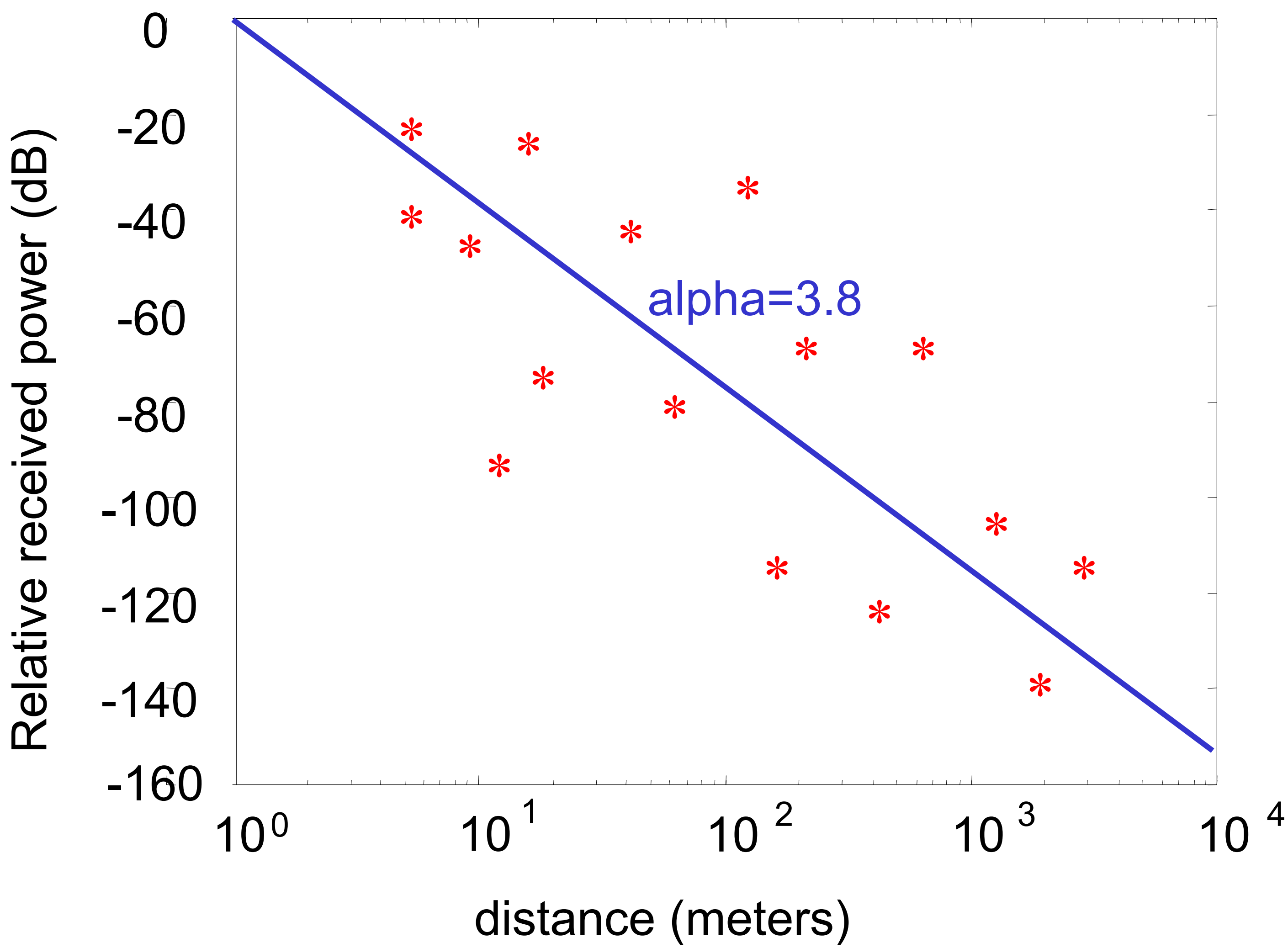
Attenuation v. distance



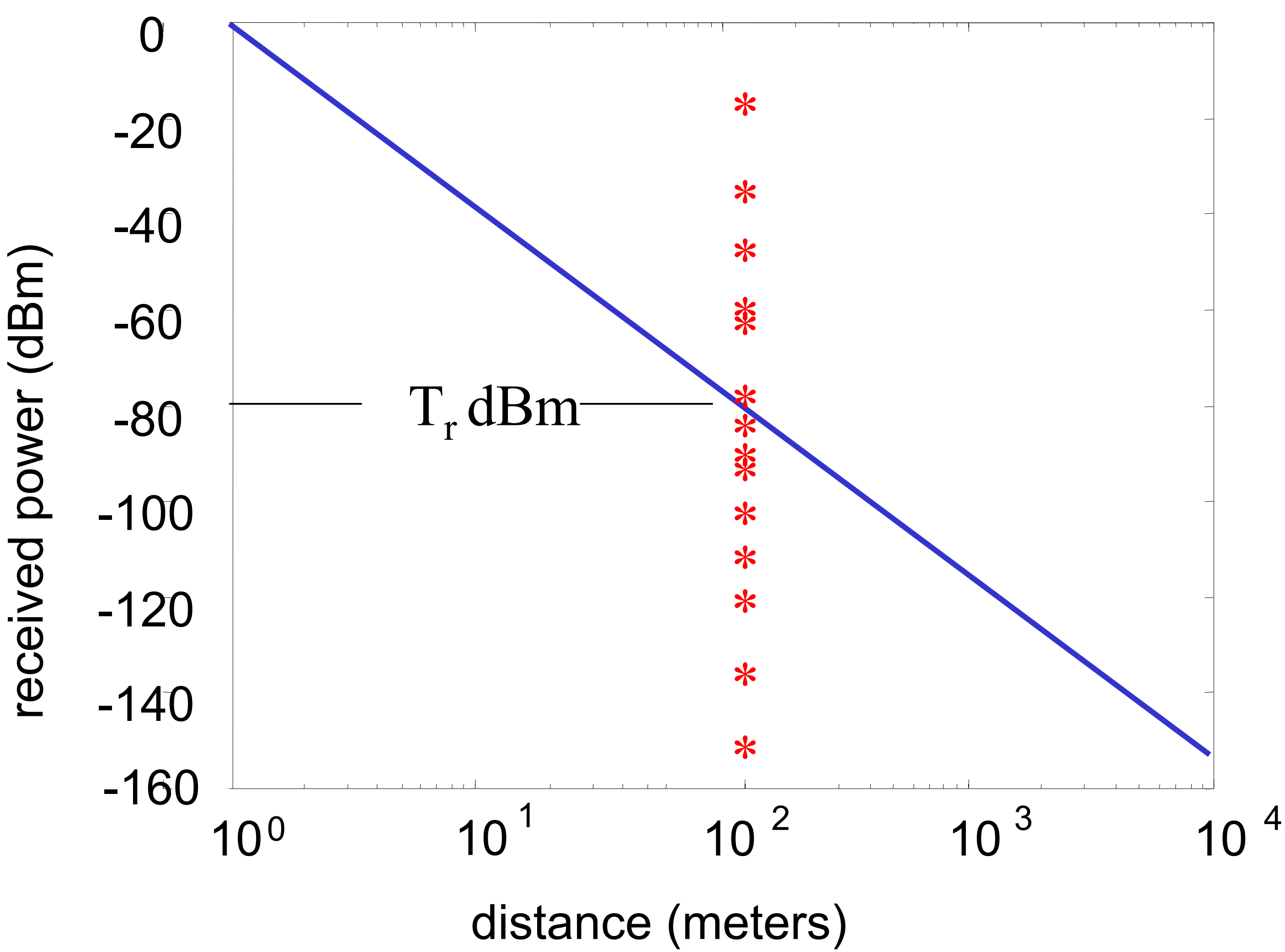
Attenuation v. distance



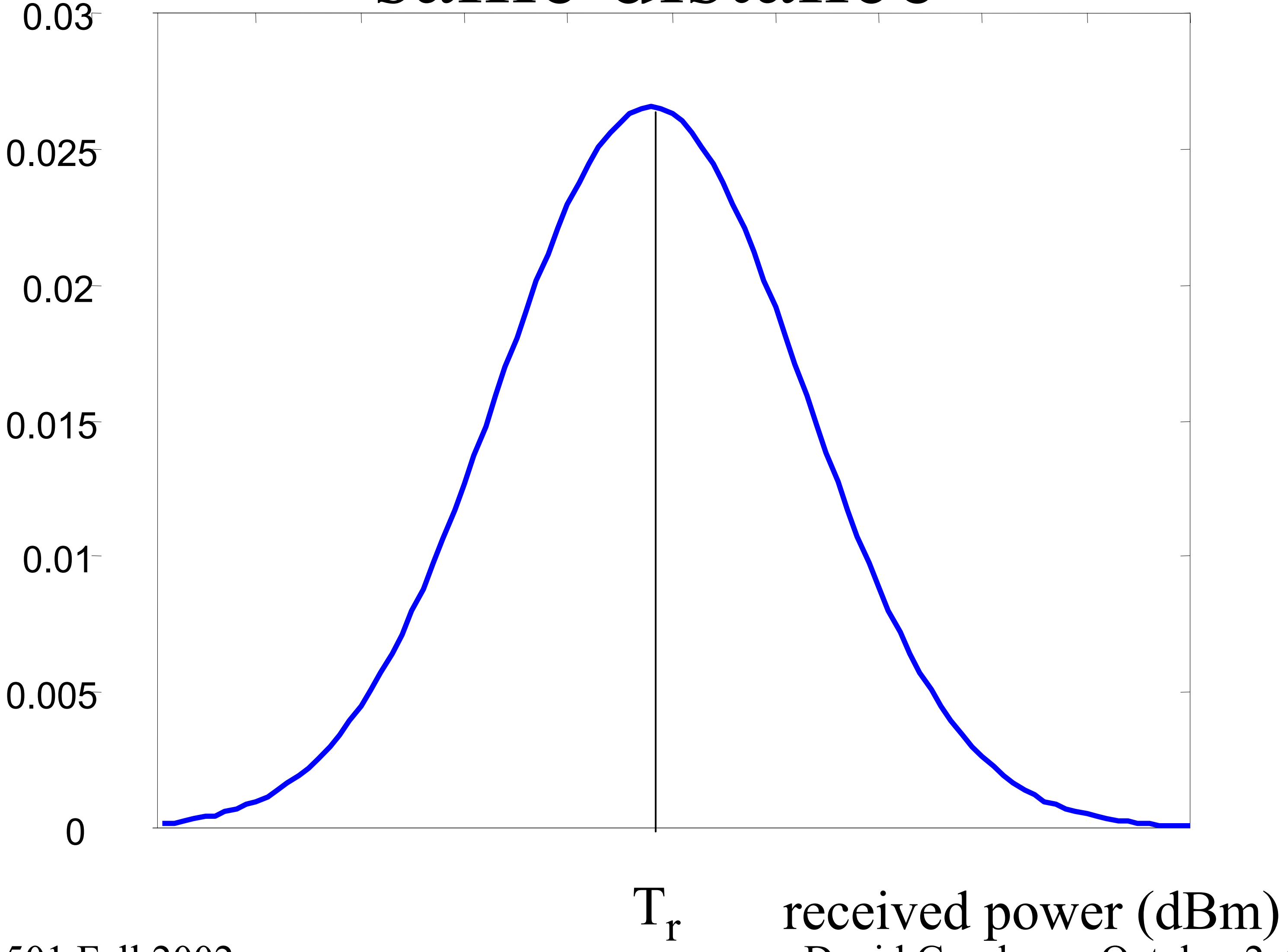
Attenuation v. distance



Attenuation v. distance



PDF of received power (dB) at same distance



Shadow fading model

- The random variable T is a power measurement (in dBm) taken at a specified transmitter-receiver distance
- T_r is the average of all power measurements (in dBm) taken at the same distance.
- T is a normal random variable with expected value $E[T]=T_r$ and standard deviation σ dB

$$f_T(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\left[t - T_r\right]^2}{2\sigma^2} \right].$$

- σ is the shadow fading standard deviation

Attenuation and shadow fading models

- Received power is a function of distance

$$P_r = \frac{\text{const}}{d^{\alpha}} \quad \text{watts}$$

- Engineers use decibel measures

  T_r=10log10(1000 P_r) dBm

- T_r is an average – the actual power in dBm at distance d meters is a normal random variable T, with E[T] = T_r and Var[T]=σ^2

Shadow fading model

- T dBm is a normal random variable
  - Expected value T_r dBm depends on transmitter-receiver distance.
  - Standard deviation σ depends on environment, "typically" σ = 6 – 8 dB (depending on uniformity of features)
- Corresponding to T is a power measurement in watts
- $T = 10\log_{10}(1000P)$ $P = 0.001 \times 10^{(T/10)}$ watts
- P watts is a a log-normal random variable

Shadow fading is slow fading

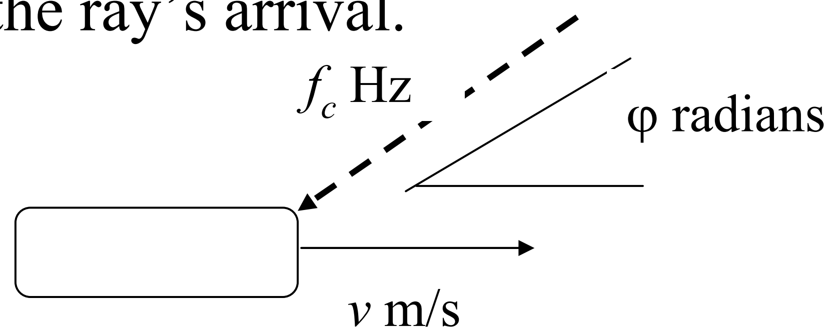
- The randomness in power measurement T is due to the unpredictable effects of terrain (hills, buildings, roads, outdoors; walls, furniture, people indoors)
- (Prof Bertoni at Poly is world famous for deriving deterministic models based on geometric diffraction theory.)
- T changes slowly as you move from one place to another (at constant transmitter-receiver distance).

Spatial correlation of slow fading

- Probability experiment
  - Measure the power in (dBm) at two random locations both d meters from a transmitter. The measurements are sample values of two normal random variables, T_1 and T_2:
  - both are $N(T_r,\sigma^2)$.
- The covariance of T_1 and T_2 depends on the distance x meters between the measurements
  - $Cov(T_1,T_2)=\sigma^2 exp(-x/x_{shadow})$
  - Correlation distance x_{shadow} meters is a function of environment depending on spacing of features. In cities x_{shadow} is related to street grid. A "typical" value is 45 m.

Effect of mobility on a received sine wave

•Transmit a sine wave with carrier frequency f_c Hz, wavelength λ meters. Receive one ray of the sine wave at a terminal moving With velocity v meters/second in a direction ϕ radians relative to the angle of the ray's arrival.



•Motion causes a Doppler shift to frequency $f_c + f_d \cos\phi$ Hz, where Doppler frequency is $f_d = v/\lambda$ Hz.

•A moving transmitter has the same effect as a receiver in motion.

Effect of mobility

•Due to scattering many rays arrive, each at its own angle and each with its own Doppler shift. They add in the receiver to produce a composite signal with the individual rays at frequencies centered at f_c.

•As the terminal moves, the components reinforce each other at some locations and cancel at others. Therefore the signal envelope fluctuates. The ups and downs occur at position changes on the order of one wavelength.

•The sum of the rays can be modeled as a narrowband signal centered on f_c Hz with waveform

$$r_S(t) = A(t)\cos 2\pi f_c t + B(t)\sin 2\pi f_c t = C(t)\cos\left[2\pi f_c t + \phi(t)\right]$$

•The in phase and quadrature components A(t) and B(t) are sums of many random variables. The central limit theorem suggests they are both normal with E[A(t)]=E[B(t)]=0.

Probability facts

- Given independent identically distributed normal random variables A(t) and B(t) and the derived random variables:

$$C(t) = \left\{ [A(t)]^2 + [B(t)]^2 \right\}^{1/2}$$

$$\phi(t) = \tan^{-1}[B(t) / A(t)]$$

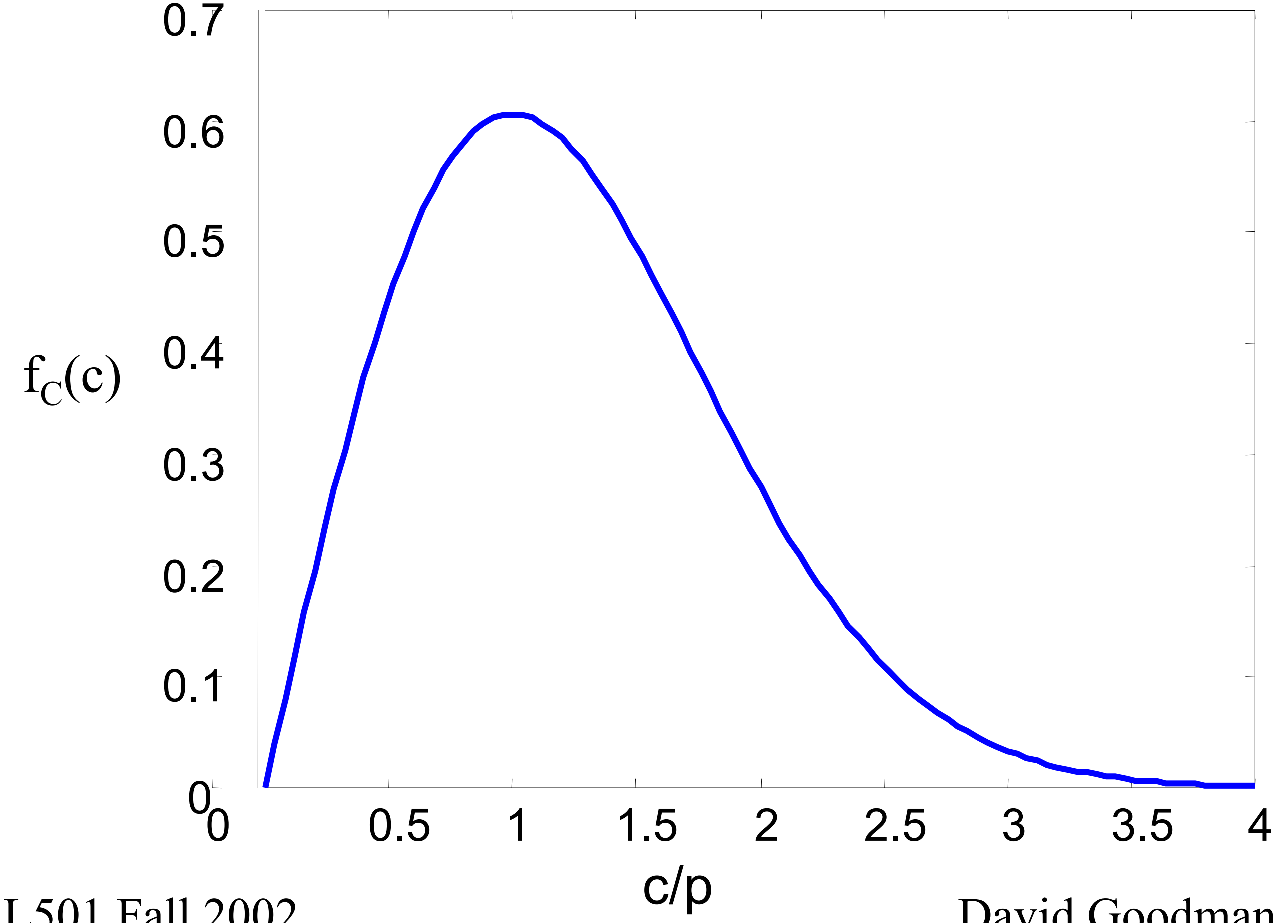
- C(t) is a Rayleigh random variable with $E[C^2(t)] = 2p$ and $\phi(t)$ is a uniform random variable over $(0,2\pi]$.

- $p=E[A^2(t)]= E[B^2(t)]$

Rayleigh PDF

$$f_c(c) = \frac{c}{p}\exp\left(-\frac{c^2}{2p}\right), \quad c \ge 0$$

$$= 0, \quad c < 0$$

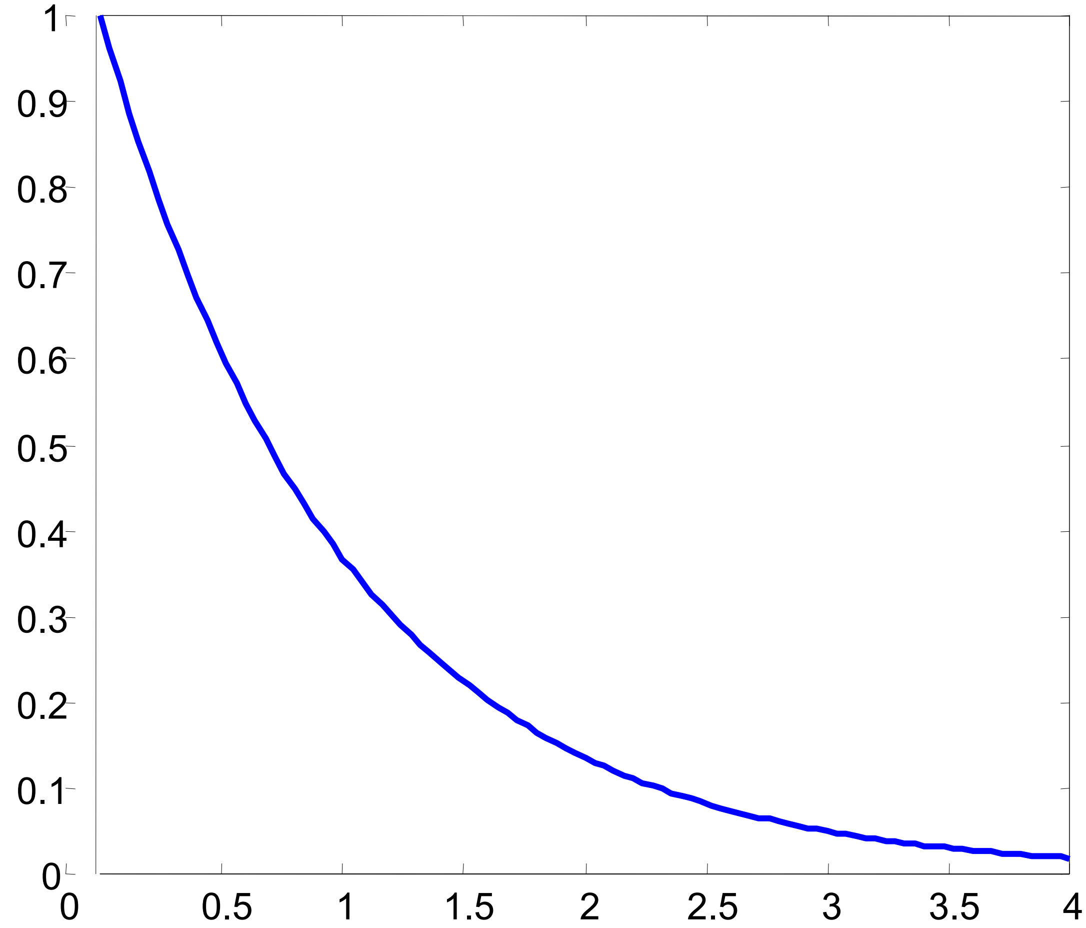


Probability facts

$$S(t) = 0.5[C(t)]^2$$

is a sample function of an exponential Random variable with E[S(t)]=p

$$f_s(s) = \frac{1}{p}\exp\left[-\frac{s}{p}\right], \quad s \geq 0$$
$$= 0, \quad s < 0.$$



Engineering relevance

- C(t) is the signal envelope **(Rayleigh)**
- $\phi(t)$ is the phase **(Uniform)**
- S(t) is proportional to the instantaneous power (if the proportionality constant =1, S(t) is the instantaneous power) **(Exponential)**
- E[S(t)]=p is the average power **(log normal)**
- p is a sample value of the random variable

$P=10^{(T/10)}$ described in the shadow fading context

Effect of a direct ray

- The Rayleigh model assumes that the received signal consists of many scattered random components, all with the same statistics.
- In some environments (especially indoors) there is a direct line of sight between transmitter and receiver with the rays along this line much stronger than others.
- The Rice probability model (a generalization of Rayleigh) describes this phenomenon.

Rice probability model

Direct ray: sine wave with magnitude D volts, phase θ radians

Envelope and phase of composite

$$C(t)=\left\{\left[A_S(t)+D\cos\theta\right]^2+\left[B_S(t)-D\sin\theta\right]^2\right\}^{1/2}$$

$$\phi(t)=\tan^{-1}\left[\frac{B_S(t)-D\sin\theta}{A_S(t)+D\cos\theta}\right].$$

Rice proability density function

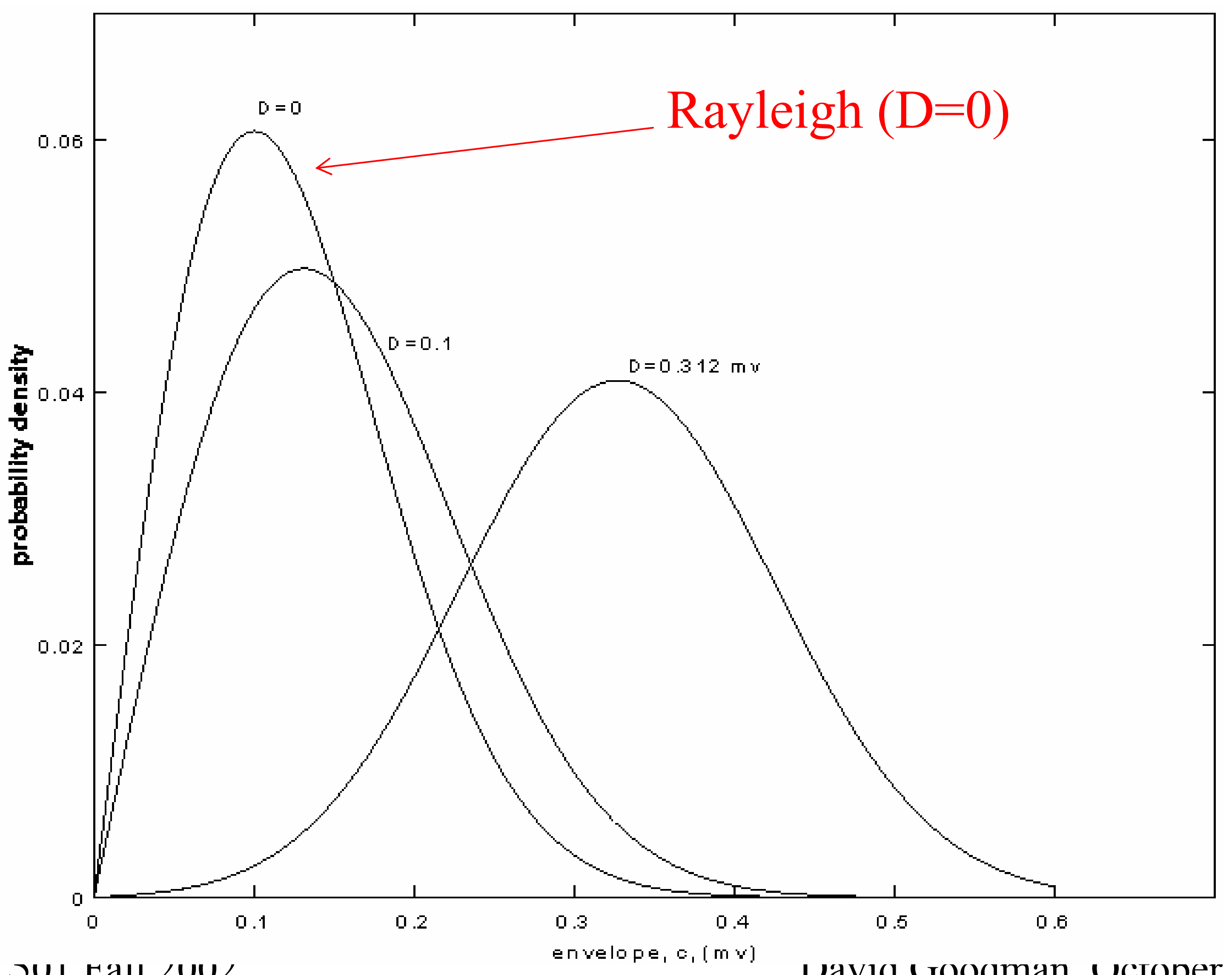
$$f_c(c)=\frac{c}{p}\exp\left[-\frac{c^2+D^2}{2p}\right]I_0\left[\frac{cD}{p}\right],\quad 0\le c\le\infty,$$

$$=0,\quad c<0.$$

Bessel function definition

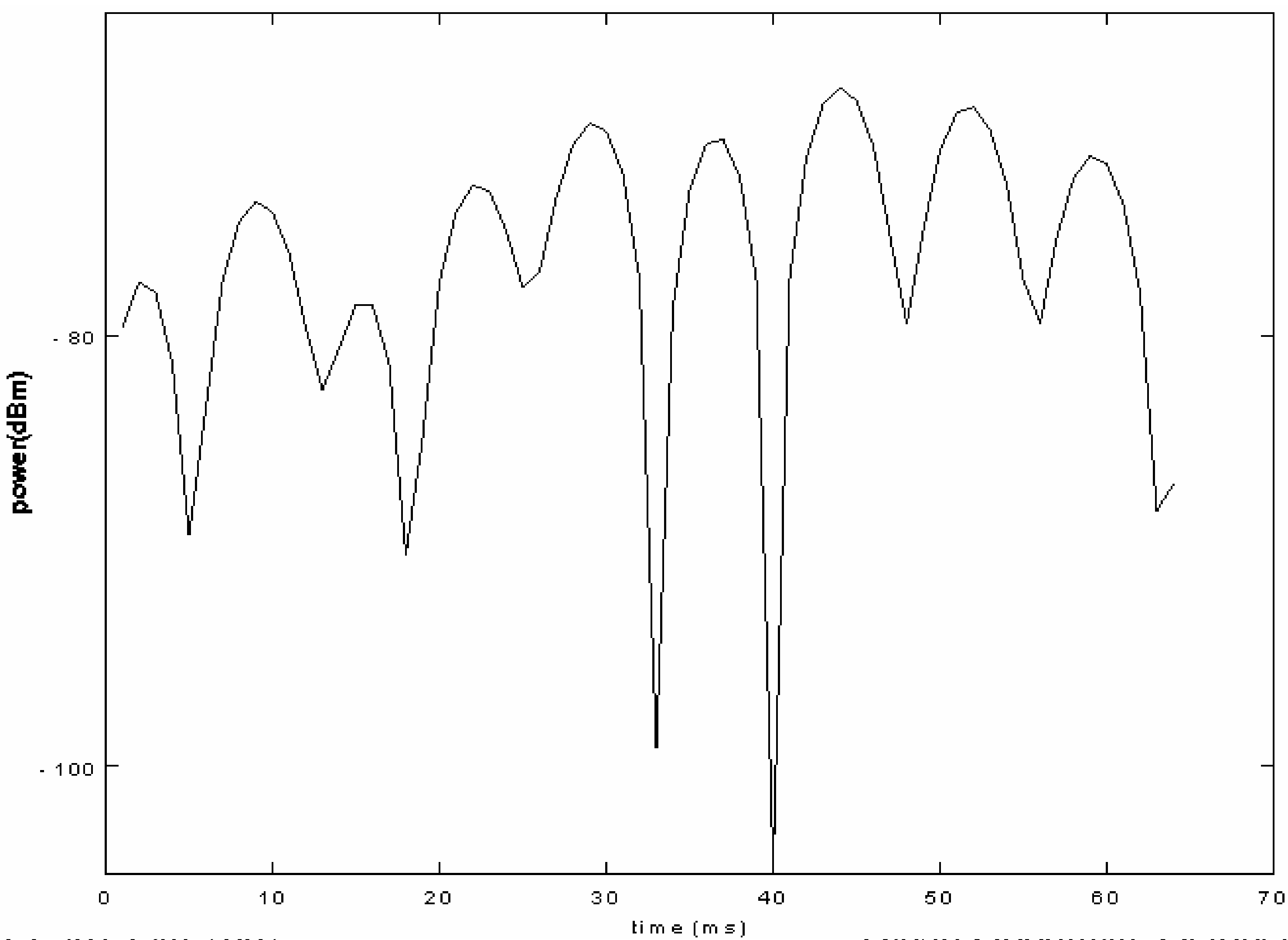
$$I_0(x)=\frac{1}{\pi}\int_0^{\pi}\exp\left(x\cos\theta\right)\mathrm{d}\theta$$

Rice pdf examples



Rayleigh fading power variations

power (in dB) v. time



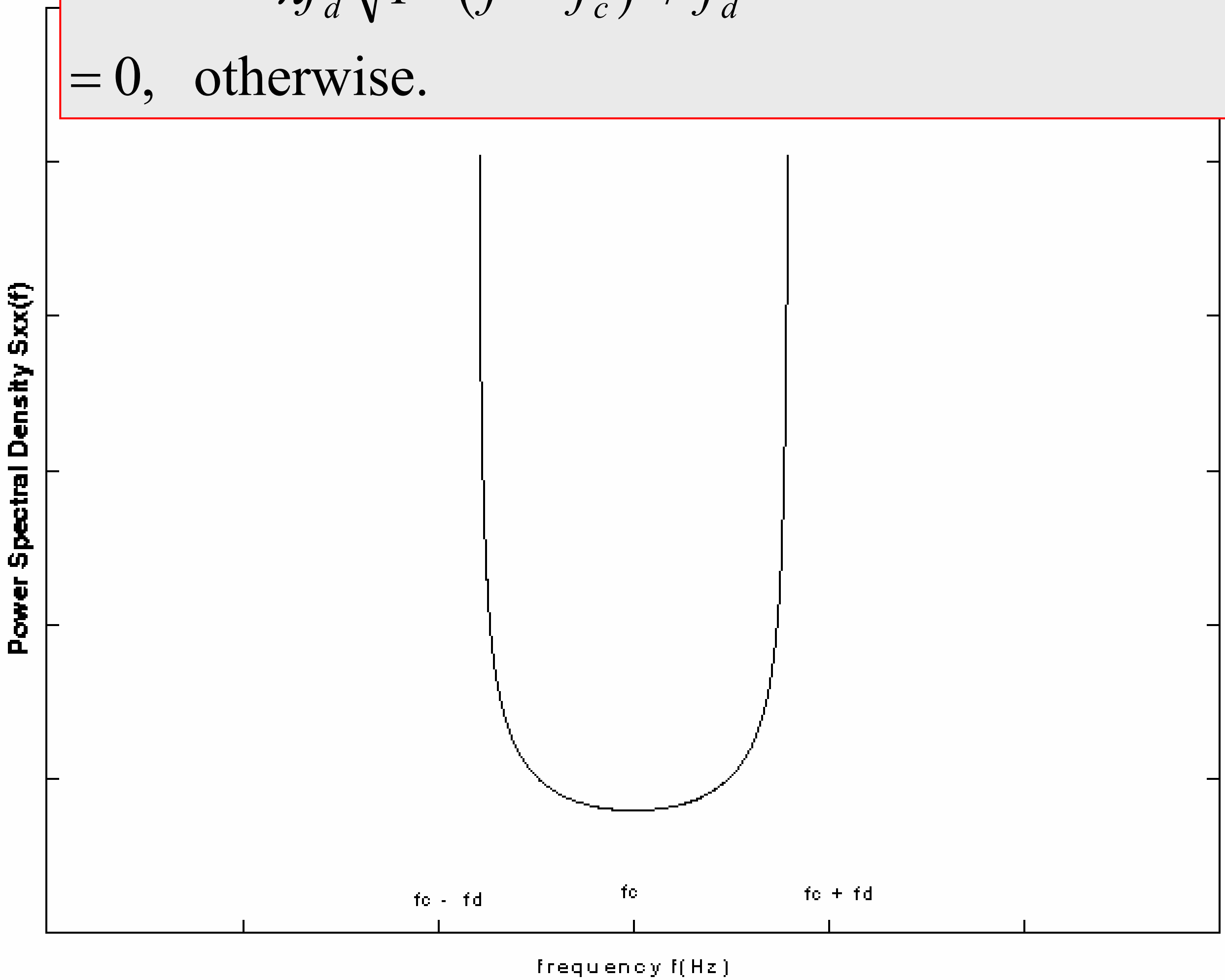
Functions and paramenters describing variations with time

- Autocorrelation function of signal $R_{rr}(\tau)$
- Power spectral density $S_{rr}(f)$
- Number of fades per second
  - below R volts $N(R)$
- Fade duration $\tau(R)$
- Interfade duration $\tau'(R)$

Power spectral density

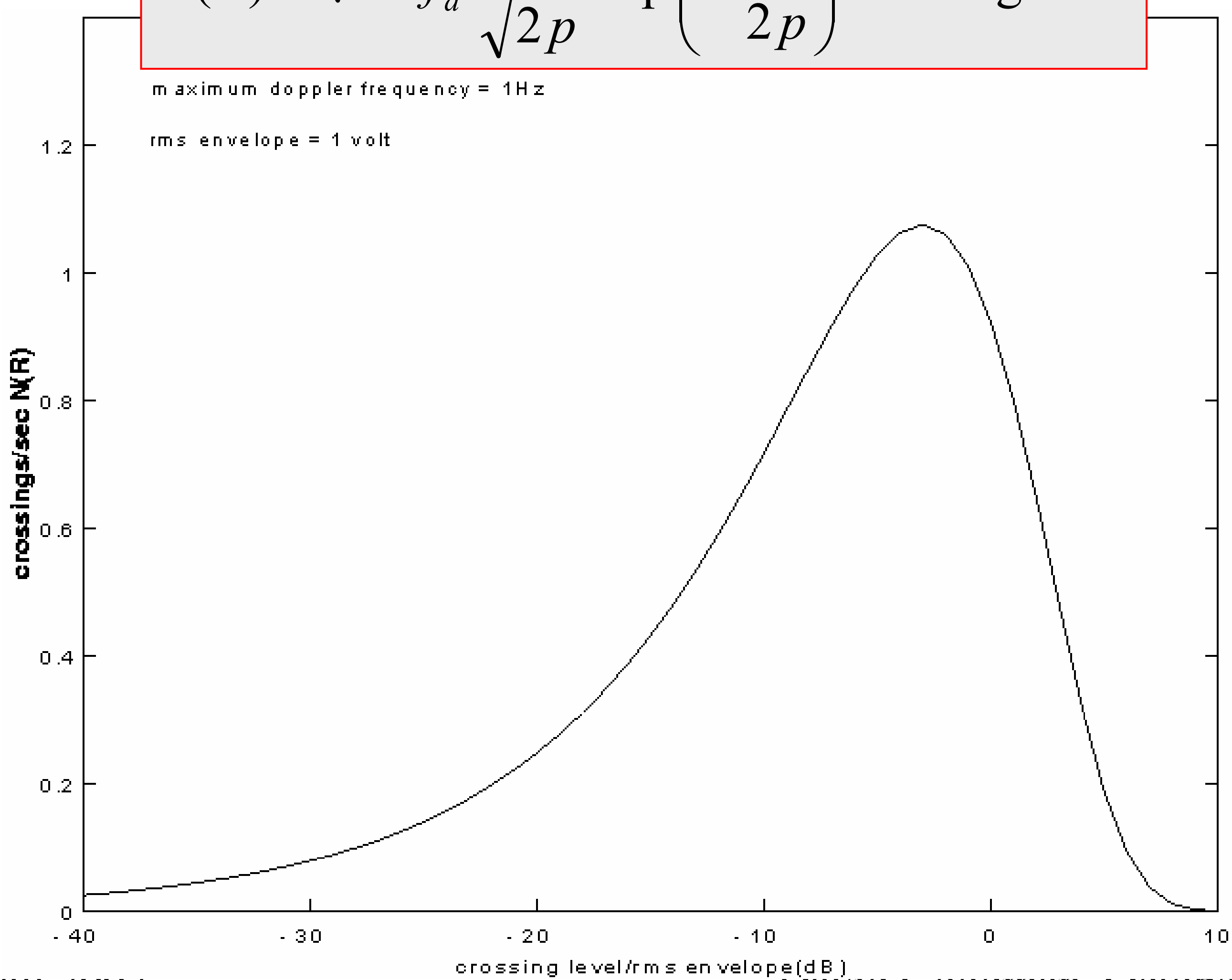
$$S_{rr}(f) = \frac{1}{\pi f_d \sqrt{1 - (f - f_c)^2 / f_d^2}}, \quad f_c - f_d < f < f_c + f_d$$

$$= 0, \quad \text{otherwise.}$$



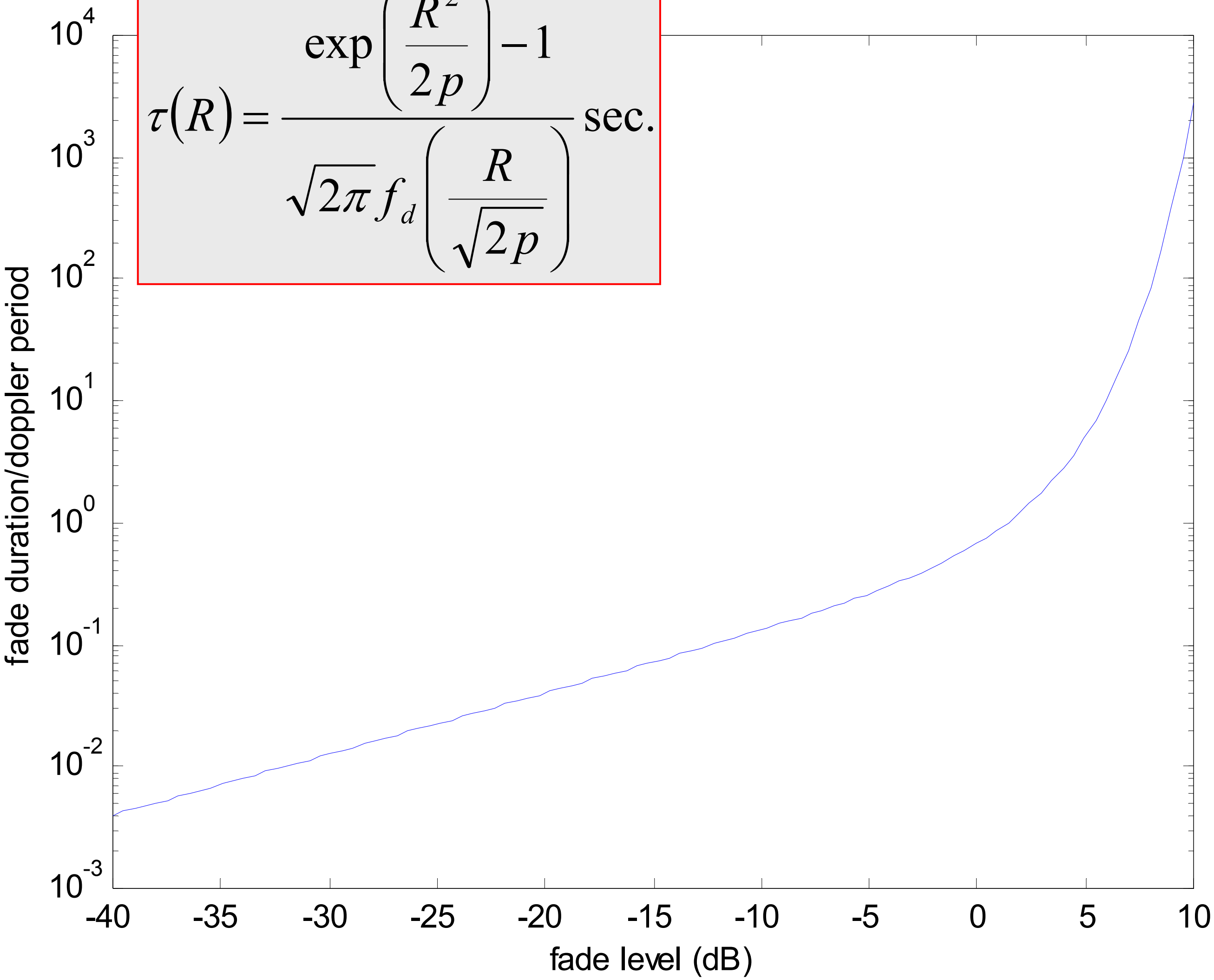
Fade rate
(depends of fade depth)

$$N(R) = \sqrt{2\pi}\, f_d \frac{R}{\sqrt{2p}} \exp\left(-\frac{R^2}{2p}\right) \text{crossings/sec.}$$



Fade duration
how long on average does C remain <R?

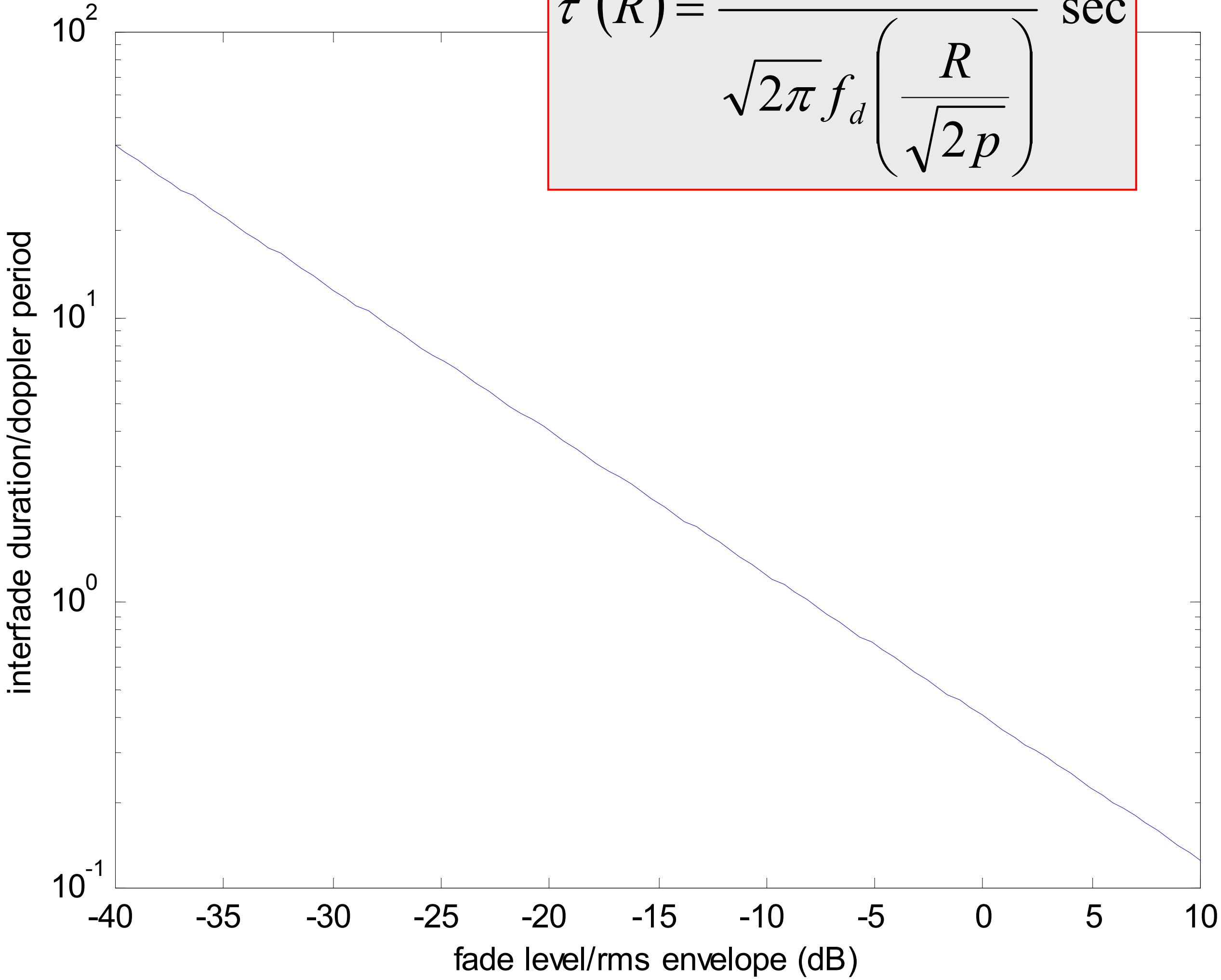
$$\tau(R) = \frac{\exp\left(\frac{R^2}{2p}\right) - 1}{\sqrt{2\pi} f_d \left(\frac{R}{\sqrt{2p}}\right)} \text{ sec.}$$



Interfade duration

how long on average does C remain >R?

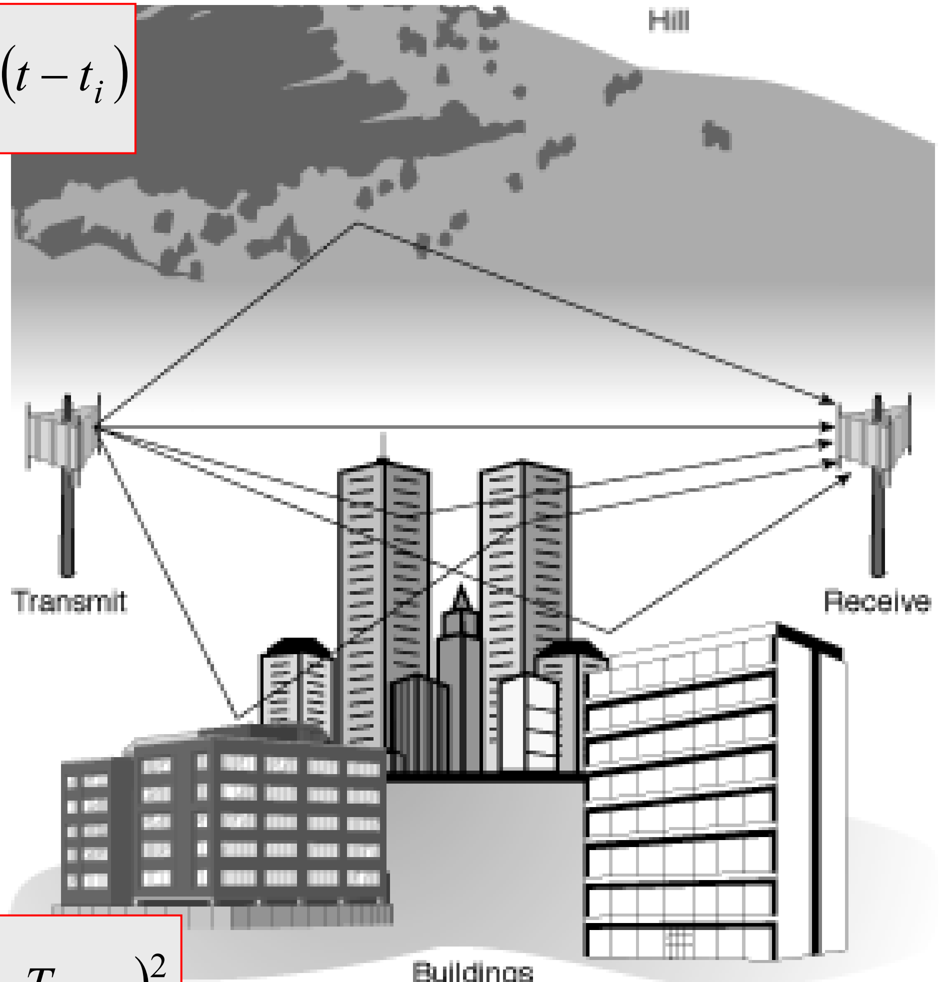
$$\tau'(R) = \frac{1}{\sqrt{2\pi}\, f_d \left(\frac{R}{\sqrt{2p}} \right)} \text{ sec}$$



Multipath

$$r_M(t) = \sum_{i=1}^{N} a_i r_S(t - t_i)$$

$$T_{\max} = t_N - t_1 .$$



$$T_{\text{rms}}^2 = \sum_{i=1}^{N} a_i \left(t_i - T_{\text{mean}}\right)^2$$

$$T_{\text{mean}} = \sum_{i=1}^{N} a_i t_i$$